The 8th International Scientific Conference
"DEFENSE RESOURCES MANAGEMENT IN THE 21st CENTURY"
Braşov, November 14th 2013

DEFENSE CAPABILITIES PLANNING USING GAME THEORY

Lt Col eng. Marian Nedelcu

The 8th LAROM BRIGADE/Romanian Land Forces

Abstract:

There are new trends and concepts in defense capability planning. All of them try to develop models capable to provide efficiency in resources allocation and obtain the maximum response to the national security.

Defense capability planning using game theory falls within a much broader category of conflict interaction that also includes economics competitions, election campaigns, industrials disputes, lawsuits.

In this paper I develop a model, based on mathematical tools provided by game theory, to study how the defense planners have to react to a given sets of threats, and how this treats is changing as a response to the capability structure and level of readiness.

The national security is viewed as a strategic defender who defends a system of independent components against attacks of strategic attackers. The reliability of system's components depends on how strongly it is defended and attacked.

Key words: game theory, defense capability, conflict, the contest success function

1.Introduction

Game theory provides a number of analytical tools designed to assist us in more comprehensive understanding of phenomena that we observe when decision-makers interact. A game describes the strategic interactions between players who act through their interests with the consciousness that their action affect each other. The basic entity in all game theoretic models is the player. A player may be understood as an individual, group of individuals or any kind of organization, even countries or alliances who face to make decisions.

In order to describe a theoretic game we need to specify four essential elements: players, actions, payoff and information. Rasmussen refers to these by acronym PAPI [3].

To develop a model based on game theory capable to describe an optimal defense resource allocation, and identify the planning "game" elements, it is needed to bring out the defense capability definitions, and see what this implies.

The Australian Defense Force defines the defense capability as "*the power to achieve a desired operational effect in a nominated environment, within a specified time, and to sustain that effect for a designated period.*" [4] This comprises the combined effect of multiple inputs such as: personnel, organization, training, major systems, supplies. The Department of Defense of US defines as a military capability "*the ability to achieve a desired effect under specified standards and conditions through combinations of means and ways to perform a set of tasks*" [CJCSI/M 3010 series]. It includes four major components: force structure, modernization, readiness, and sustainability.

Both definitions are built around the notion of “effect”. This leads us to the question “what decisions can be taken to maximize the general security effect, having design certain capabilities to encounter certain threats, under the pressure of limited resources at disposal (such as the allocated defense budget)?”

The model developed in this paper is focused on allocations of the available defense resources over the assumed capabilities in order to achieve the best response to the national security. I consider as a game the competition between strategic decision-makers involved in defense resource management and the threats to the national security.

2. The model formulation

In the following game model we have, on the one hand, the main player with fictitious name “defender”, who tries to defend as much as possible a series of valuable targets. On the other hand we have a series of “attackers” who tries to attack the targets and determine as much loses as possible.

The notion of “attacker” does not want to define a specific country or terrorist organization that interacts with national security; it takes into consideration the effect of its action as a “threat” against national security. This threat is evaluated in terms of probability of occurrence and the level of impact over the target on which is directed. I assume that the attackers (as well as their generated threats) are neither static, or fixed, nor immutable.

When I refer to a target I mean a complex system which can be conceived to have economic value, human value, and/or symbolic value, and could generate an interest for a potential aggressor. As example, the target such as The U.S Statue of Liberty has substantial symbolic value, and no human value. I don’t want to insist with more details what a target comprises and how this can be evaluated. For my purpose I take into consideration a theoretical target which has a different value from defender and attacker perspective.

The first and foremost approach to modeling a player’s interest is utility theory. This theoretical methodology deals with measuring the degree of preference across a set of available options. The purpose of this paper is not to evaluate the targets, but we need to set the theoretical values for each target, measured in money. Further research will explore in details the ways of evaluation.

I denote with “v_i” a total value of the target “i” (economic, human and symbolic) from the defender point of view. From the attacker “j” perspective, V_i^j represent the total value of the target “i”.

The functionality or successful operation of each target depends on the relative investments in defense versus attack. The defender seeks functionality of the system while the attacker seeks non-functionality. [9]

The approach allows analyzing the phenomenon from both the defender and attackers’ point of view. [9]

The defender and attackers are considered having both strategically thinking and be capable to adapt optimally own action to the opponent action by spending defensive/offensive resources for each target.

In order to assure the reliability of a target “i” the defender assigns a capability which needs having spent the resources “x_i” to have assured a certain level of readiness. The total cost of allocated resources is constraint by the defense budget B_d.

$$\sum_{i}^{n} x_i = B_d \ (1)$$

where:

- n – the number of the capabilities;
- B_d – the defense budget;

In the same time each attacker tries to allocate resources y_i^j in such ways to decrease the reliability of each target, also being constrained by the budget.

$$\sum_{i}^{n}\sum_{j}^{m} y_i^j = B_a \ (2)$$

where:

- m – the number of the attackers;
- B_a – the budget of the attackers;

Tullock (1980) has introduced in game theory the notion of rent-seeking contest in which players compete for a rent of a given value. The fundamental notion of a contest is the contest function success (CFS) which measures the probability for each player to gain the prize. Tullock has come with two forms of this function: one of them is based on the ration of the effort of each player, and another based on difference between the players' resource allocation.

The probability to have success in defending the target *"i"* is given by the relation:

$$p_i = \frac{(x_i)^{\lambda_i}}{(x_i)^{\lambda_i} + \sum_{j}^{m}\left(y_i^j\right)^{\lambda_j}} \ (3)$$

- $\lambda_i \geq 0 \ i = 1 \cdots n$ is the cost-effectiveness of each capability;

Employing the difference form instead, we can write the contest function success using the logistic function as following:

$$p_i = \frac{e^{\lambda_i x_i}}{\sum_{j}^{m} e^{\lambda_j y_j}} \ (4)$$

or

$$p_i = \frac{1}{\sum_{j}^{m} e^{\left(\lambda_j y_i^j - \lambda_i x_i\right)}} \ (5)$$

Te ratio form is used in confrontations which take place under "idealized" condition such as: full information, unflagging weapons effectiveness. In contrast, the difference form is used when information is imperfect, and the victorious player is a subject to fatigue and distraction. [8]

The aim of defender is to save as much as is possible from the total value of the defended targets, through resource allocations for each developed capability, knowing the attackers' efforts (resource allocation) for each targets.

The total value expected to be saved by defender if a confrontation with the attacker will occur is:

$$S\left(x_i, y_i^j, v_i, V_i^j\right) = \sum_i^n p_i \cdot v_i \quad (6)$$

The objective function for defender is to maximize the total value expected to be saved:

$$\max_{x_i} S\left(x_i, y_i^j, v_i, V_i^j\right) \quad (7)$$

From the perspective of attackers the contest functions success can be written as following:

$$q_i^j = \frac{\left(y_i^j\right)^{\lambda_j}}{\left(x_i\right)^{\lambda_i} + \left(y_i^j\right)^{\lambda_j}} \quad (8)$$

or using the logistic function (the difference form):

$$q_i^j = \frac{e^{\lambda_j y_i^j}}{e^{\lambda_i x_i} + e^{\lambda_j y_i^j}} \text{ for each attacker } j = 1 \cdots m \quad (9)$$

The total value expected to be lost by defender if a confrontation with the attacker will occur is:

$$L^j\left(x_i, y_i^j, v_i, V_i^j\right) = \sum_i^n q_i^j \cdot V_i^j \text{ for each attacker } j = 1 \cdots m \quad (10)$$

The aim of the attackers is to cause as much as possible damages (lost value) to the defender (targets).

$$\max_{y_i^j} L^j\left(x_i, y_i^j, v_i, V_i^j\right) \quad (11)$$

3. THE SOLUTION

Let's suppose a virtual country, XLand (dfender), in a security environment which has to face with a series of threats from a virtual enemy, YLand (attacker). The Ministry of Defense has to develop a series of capabilities, with a restraint budget, in order to counter these threats.

In the process of capability based planning it was assumed as necessary three types of capabilities: A - land defense, B- air defense, and C – maritime defense. Each of these capabilities is developed to defense, against the virtual enemy, three virtual independent targets (having human value, economical value and also symbolic value): T1 - land territory, T_2 – airspace, T_3 – maritime territory.

It is assumed both "defender" and "attacker" have different evaluation of three targets as described in Table 1. Their actions (defending and attacking) against three targets are limited by the budget allocation: B_d = \$300 billion – the defender's budget, B_a = \$400 billion – the attacker budget.

The defense planning specialists evaluate the cost-cost effectiveness of both own capabilities, and attacker capabilities through following parameters: $\lambda_d = 0.3$, and $\lambda_a = 0.1$.

To estimate the probability of success both attacker and defender use the difference form of CFS.

Taking into account all above assumptions and evaluations, and with the help of Excel solver, it can be found out that the model rapidly converge to the Nash's equilibrium, and there is a single optimal solution in resource allocations to defend/attack the targets for both defender and attacker (Table 2). This solution assure the "maximum saved value" – ***4662 billion dollars*** (from defender's perspective) and "the maximum lost value" – ***2032 billion dollars*** (from attacker's perspective), if a conflict between defender and attacker occurs. Any others allocation of resources will conduct

Capability	Targets	Defender's target evaluation (billion dollars)	Attacker's target evaluation (billion dollars)
A. – Land Defense	T1. – Land Territory	3000	2800
B. – Airspace Defense	T2. – Airspace	2000	2500
C. – Maritime Defense	T3. – Maritime Territory	1500	1700

Table 1

Capability	Targets	Defender's resource allocation (billion dollars)	Attacker's resource allocation (billion dollars)
A. – Land defense	T1. – Land Territory	134	155
B. – Airspace Defense	T2. – Airspace	94	145
C. – Maritime defense	T3. – Maritime Territory	72	100
TOTAL RESOURCE ALLOCATION		**300**	**400**

Table 2

4. Conclusion and future research directions

This model, developed based on game theory, can become a powerful tool for defense planning makers who need to design the structure of resource allocation over the capabilities.

The major problem faced was in defining the elements of the games in such ways in which the equilibrium can be reached, and provide a solution to defense capability planners. The Rasmunsen states that ***"lack of a unique equilibrium is a major problem in game theory"*****[3]**.

Even some parameters of the game are hard to be evaluated (the value of the targets, the cost effectiveness parameters) the model can provide a structure of resource allocation over capabilities at disposal. There is no so important to know the absolute values of the parameters. It is very useful if we have relative evaluations, and if we could establish rations between them within a certain tolerance field.

Also, the results can show that it no necessary to know how the attacker allocates resources over his capabilities. To take an optimum decision it is enough if it is known the budget and the nature of attacker's capabilities (in order to estimate the cost-effectiveness parameter) and estimate the threats (the value of the targets from attacker perspective).

This simplified model can be first step in a more complex analysis of defense capability planning based on resource allocation.

Interesting future research can include: (a) sensitivity analysis and determine the important parameters which have the most influence over the resource allocation; (b) how the cost-effectiveness parameter influence resource allocation, and the probability to win against attacker. What are the circumstances under which the planners decide to acquire a new capability instead to maintain an older one? (c) develop a more complex model with multi-purpose capabilities in an security environment with more independent and/or dependent enemies; (d) consider the enemies who act both strategically and non-strategically.

Acknowledgements

This paper is an outgrowth of my studies during the Postgraduate Defense Resources Management Course for Senior Officials in Regional Center of Defense Resources Management, Brasov.

Here, I had the opportunity to open my mind and deal with the newest concepts used in the world of defense planning. I am grateful to all teachers that I have met here, but especial thoughts to PhD Maria CONSTANTINESCU who polished my knowledge in Capability-Based Planning and Planning Programing Budgeting and Evaluating System, and to COL Phd Daniel SORA, the person who encouraged me to see the subtleties of using mathematical tools such as Game Theory, and Decision Theory in decisional process.

Definitions:

Reliability is the capacity of system to carry out and keep its functions in regular circumstances, as well as aggressive or unanticipated situation. (Wikipedia)

Contest is a game in which the players compete for a prize by exerting their effort so as so to increase their probability of winning [1];

Contest Success Function (CSF) provides each player's probability of a winning for any given level of efforts [1];

Nash's equilibrium (in economics and game theory) is a stable state of a system involving the interaction of different participants, in which no participant can gain by a unilateral change of strategy if the strategies of the others remain unchanged. (Oxford dictionary)

References:

[1] Stergios Skaperdas, 1996. *Contest success functions*. Economic Theory 7, 283-290 (1996);

[2] Martin J. Osborne and Ariel Rubinstein. *A Course in Game Theory, electronic version*;

[3] Eric Rasmunsen. A *introduction in game theory, Game and Information*. Third edition (2001);

[4] Australian Government, Department of Defense. Defense Capability Development Manual (2006);

[5] Victor Richmond R.Jose - Georgetown University, Jun Zhuang – University at Buffalo. *Technology Adoption, Accumulation, and Competition in Multi-period Attacker Defender Games.* Military Operations Research, V18 N2 2013;

[6] Xiaojun Shan, Jun Zhuang – University at Buffalo. *Hybrid defensive resource allocations in the face of partially strategic attackers in a sequential defender–attacker game.* European Jurnal of Operational Research 228 (2013);

[7] Tullock, G., 1980. *Efficient rent seeking*. In: Buchanan, J.M., Tollison, R.D., Tuliock, G. (Eds.), Toward a a theory of the rent-seeking society. Texas A and M University Press, College Station, pp, 97-112;

[8] Jack Hiorshleifer. *Conflict and rent-seeking success function: Ratio vs difference models to relative success*. Public choice 63: 101-112, 1989;

[9] Kjell Hausken. *Protecting Complex Infrastructure Against Strategic Attackers*. Faculty of Social Sciences, University of Stavanger, Norway. 2007

Appendix 1

DEFENDER

Budget	**300**	
Resource allocation (billion dollars)		**Probability of success**
Land Defence	**134**	0,72
Airspace Defance	**94**	0,70
Maritime defense	**72**	0,69
TOTAL RESOURCES	300	
Cost-efectiveness	0,3	

Target	**Evaluation (bilion dollars)**	
	Defender persepective	**Attacker perspective**
Land	3000	2800
Airspace	2000	2500
Maritime	1500	1700

The expected saved value (bilion dollars)	**The expected loss value (bilion dollars)**
4622	**2032**

ATTACKER

Budget	**400**	
Resource allocation (billion dollars)		**Probability of success**
Land Attack	**155**	0,28
Airspace Attack	**145**	0,30
Maritime Attack	**100**	0,31
TOTAL RESOURCES	400	
Cost-efectiveness	0,1	

10	The variables (changing cells)
30	The constrain
40	The target